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“Variational analysis of fluid equations with help from computational optimization”

Various types of statements about dynamical ODEs and PDEs can be proved by finding 'auxiliary functions', meaning scalar-valued functions whose time evolution obeys certain inequalities that imply the desired result. One type of auxiliary function is a Lyapunov function, which satisfies inequalities guaranteeing that the function is positive and decreasing in time, and which implies nonlinear stability of a particular state. Other types of auxiliary functions, subject to different inequalities, imply other statements such as bounds on time-averaged or instantaneous values of a chosen quantity of interest. In many cases involving the Navier–Stokes equations and other fluid models, all auxiliary functions used in the past have been quadratic integrals, and the relevant inequality conditions can be checked by solving a quadratic variational problem. When auxiliary functions are quadratic integrals, showing stability amounts to the 'energy method', and bounding quantities of interest amounts to the 'background method'. I will explain why it is technically hard to generalize to auxiliary functions that are not quadratic integrals, in part because one confronts harder variational problems that must be relaxed. I will present one way of meeting these difficulties that involves the computational solution of relaxed variational problems using a type of convex optimization called semidefinite programming. I will show two examples in which this approach has been applied: global stability of Couette flow beyond the energy stability threshold, and bounds on mean energy in the 1D Kuramoto–Sivashinsky equation.