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"On the Construction of Stable Pairings of Scalar and Vectorial Multi-scale Finite Elements in 3D for Discrete L2-Differential Complexes"

Simulations of complex systems such as the atmosphere or the ocean require coarse meshes due to computational constraints. This is particularly true over long time scales. Nonetheless, capturing realistic dynamics also requires to take into account interactions with other (parameterized) physical processes across a wide range of scales that often cannot be resolved. Thus the influence of small scale processes has to be taken care of by different means. State-of-the-art dynamical cores represent the influence of subscale processes by employing heuristic coupling of scales. This, however, unfortunately often lacks mathematical consistency. As a second problem the exact preservation of balances or certain physical quantities such as energy requires finite element pairings to be "compatible", i.e., finite element projections must commute with exterior derivatives.
The aim of this work is to improve mathematical consistency of the upscaling process which transfers information from the subgrid to the coarse scales of the dynamical core while adhering to principles that keep discretizations compatible. For this we first extend the idea of adding subgrid correctors to basis functions for scalar and vector valued elements discretizing quantities in various function spaces in an abstract geometrical/homological framework.
To understand the construction of the new finite elements it is necessary to understand discrete Hodge decompositions. Discussing protoypically the issue of (fine-scale weighted) Hodge decompositions I will show that standard techniques on coarse meshes fail to find good projections in all parts of a modified de Rham complex if rough data is involved and discuss an idea of how to construct multi-scale finite element (MsFEM) correctors to scalar and vector valued finite elements and, further, how to construct stable multi-scale element pairings using the theory of finite element exterior calculus (FEEC). This can be seen as a meta-framework that contains the construction of legacy MsFEMs [Efendiev et al.,2009; Graham et al., 2012] and their construction in a unified framework.
Possible applications comprise much more than "just" ocean and atmosphere dynamics and extend to porous media, elasticity, and fluid flow as well as electromagnetism in fine-scale and high-contrast media. I will provide the necessary theoretical background in homological algebra and differential geometry, and discuss a scalable MPI based HPC implementation technique suitable for large clusters. Several computational examples will be shown. I may, if time permits, discuss some ideas from
homogenization theory to attack the problem of a proof of accuracy.