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"Convex integration and synthetic turbulence"

In the past decade convex integration has been established as a powerful and versatile technique for the construction of weak solutions of various nonlinear systems of partial differential equations arising in fluid dynamics, including the 3D Euler and Navier-Stokes equations. The existence theorems obtained in this way come at a high price: solutions are highly irregular, non-differentiable, and very much non-unique as there is usually infinitely many of them. Therefore this technique has often been thought of as a way to obtain mathematical counterexamples in the spirit of Weierstrass' non-differentiable function, rather than advancing physical theory; "pathological", "wild", "paradoxical", "counterintuitive" are some of the adjectives usually associated with solutions obtained via convex integration. In this lecture I would like to draw on some recent examples to show that there are many more sides to the story, and that, with proper usage and interpretation, the convex integration toolbox can indeed provide useful insights for problems in mathematical hydrodynamics.