

# **A Hybrid Neural Network/Finite Element Method with Applications to 3D Simulations of the Navier-Stokes Equations**

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Accurate simulation of fluid flow poses substantial challenges. This presentation explores the integration of deep neural networks with classical finite element simulations in fluid dynamics.

At high Reynolds numbers, accurate simulations in 3D settings become increasingly difficult, and the classical methods reach their limits. To address this issue, we discuss approaches to connect the finite element method with neural networks. We propose the Deep Neural Network Multigrid Solver, which combines a geometric multigrid solver with a deep neural network to overcome limitations of classical methods. This approach uses classical simulation techniques where their strengths are eminent, such as the efficient representation of a coarse, large-scale flow field. Neural networks are used when a full resolution of the effects does not seem possible or efficient.

We demonstrate the efficiency, generalizability, and scalability of our proposed approach using 3D simulations. Our focus is particularly on issues of stability, generalizability, and error accuracy. Finally, we give an outlook on the combination of hybrid methods with classical approaches to efficient simulations, particularly goal-oriented adaptivity. Overall, the approach of incorporating deep neural networks into traditional finite element approximation techniques yields new possibilities to cost efficient computational techniques for the numerical solution of nonlinear partial differential equations.