Space-time finite element geometric multigrid techniques applied to fully dynamic poroelasticity

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Space-time finite element methods (STFEMs) allow the natural construction of higher order discretizations of systems of multi-physics and offer the potential to achieve accurate results on computationally feasible grids with a minimum of numerical costs;
cf. [1, 2, 4]. Further, space-time adaptivity based on a-posteriori error control by duality concepts and multi-rate in time approaches become feasible; cf. [3]. However, the block structure and solution of the resulting algebraic systems become increasingly complex for higher polynomial degrees in space and time. We analyze STFEMs for solutions to the coupled hyperbolic-parabolic problem

$$\rho\partial_t^2 u - \nabla \cdot (C\varepsilon(u)) + \alpha\nabla p = \rho f \,,$$

$$c_0\partial_t p + \alpha\nabla \cdot \partial_t u - \nabla \cdot (K\nabla p) = g \,,$$

modeling fluid flow in deformable porous media. Recently, such type of models have attracted researchers' interest in several branches of natural sciences and technology, for instance, to elucidate circulatory diseases in the human brain. For the solving the algebraic systems, we present a robust geometric multigrid (GMG) preconditioner for GMRES iterations. The GMG method uses a local Vanka-type smoother. Due to nonlocal coupling mechanisms of unknowns, the smoother is applied on patches of elements.
The parallel implementation of the solver for arbitrary polynomial degrees in space and time in the deal.ii library (cf. [1, 2]) is addressed briefly. Numerical performance studies for challenging two- and three-dimensional benchmark settings are discussed.
This includes the convergence of goal quantities of physical interest.