Space-time DG for the linear transport equation

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We introduce the model for linear transport within the setting
for linear conservation laws, where a density distribution is transported
along a flux vector. An example is, e.g., the pollution of oil
in the groundwater.

In the first step, we recall established discretizations for this application class.
Then we consider variational space-time discretizations for the linear
transport equation with full upwind discontinuous Galerkin methods
in space and time.

Based on our convergence analysis for symmetric Friedrichs systems in
a mesh-dependent DG we construct an error estimator and show
numerically that the adaptive method is efficient. The linear system
is solved by a multigrid method in space and time, and we show
numerically that the convergence is of optimal complexity.
We observe that convergence is obtained also in case of discontinuous
solutions without regularity requirements.

Finally, we show that in case of local sources and local goal functionals the
computational domain can be restricted to a subset of the space-time cylinder
and that then a suitable parallel strategy results in a significant
reduction of the computational effect.