The Loewner Framework between compression, iteration and optimal selection

D.S. Karachalios[1], I.V. Gosea[1], and A.C. Antoulas[1,2,3]

[1]*Data-Driven System Reduction and Identification Group, Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany,*
karachalios,gosea,antoulas@mpi-magdeburg.mpg.de
[2] *Department of Electrical and Computer Engineering, Rice University, Houston, TX, USA,*
aca@rice.edu
[3]*Baylor College of Medicine, Houston, TX, USA,* antoulas@bcm.edu

Model order reduction (MOR) is a tool used to replace large and complex models of dynamical processes with smaller and simpler dimensional models, which can be easily used for complete analysis such as control, design, and simulation. The methods considered in this study are interpolation based and aim to construct reduced-order models for which the corresponding rational transfer function matches that of the original model at selected interpolation points. Moreover, the approaches we present are data-driven, in the sense that the required information is extracted from data (data points or nodes and data measurements or function values) - no original model or function is required.
The primary method of this study is the Loewner framework, as introduced in see [1]. It produces models directly from measurements in a straightforward manner. The main feature is that it provides a trade-off between accuracy of fit and complexity of model, by means of the singular value decay of the Loewner matrix.
The original Loewner framework is based on compression of the (usually large) data set in order to extract the dominant features and, in the same time, eliminate the inherent redundancies. The procedure introduced in [1] relies on a full SVD (singular value decomposition) to compress the raw data model. This might be costly to perform, especially for very large data sets.
In order to surpass this possible shortcoming, we propose a more efficient approach which replaces the SVD by a CUR decomposition. In general, a CUR approximation of a given matrix M is written in terms of three matrices C, U, and R such that C and R are composed of columns and, respectively rows, selected from the ones of the M matrix. Finally, the matrix U is chosen such that product CUR closely approximates M. We propose a modified Loewner framework, in the sense that the interpolation points are selected from the original ones, and not by means of compression. The indexing of the chosen points coincides to the indexing of the selected rows and columns of the Loewner matrix. The selection algorithm is based on the CUR-DEIM approach in [4].
We also studied the AAA algorithm, as introduced in [2]. It can be viewed as an adaptive and iterative version of the Loewner framework and aims to optimize the approximation error by means of a least squares approach. The order of the rational approximant is increased after each step for better accuracy. While these methods can often find rational approximations with a small residual norm, in order to find optimizers with respect to a weighted l2 norm with a square dense weighting matrix we will use a nonlinear least squares approach [5].
We managed to construct low order models for physical problems such as Euler-Bernoulli clamped beam model and 1D heat diffusion model. Additionally, we tested the methods for artificial examples which commonly appear in approximation theory, such as the Bessel function, the hyperbolic sine function, and the Heaviside step function.
Although a direct method, the original Loewner framework in [1] proved to be comparable (in terms of approximation quality) to the other methods which rely on optimization tools. Moreover, in the case of the Heaviside function, we were able to find rational functions via the Loewner framework, which turned out to be very close to the optimal minimax rational function (which is the solution of the fourth Zolotarev problem [3]).

References

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