The dual weighted residual method in presence of shocks

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In the dual-weighted residual method for goal-oriented error estimation residuals have to be evaluated which include the computation of derivatives. For hyperbolic problems in general the residuals are computed in the standard weak form. Potential discontinuities of the solution prohibit direct (weak) derivatives, therefore all derivatives are taken on the test function by means of partial integration. The same holds true for the dual problem. In the dual-weighted residual method, the test function of the primal residual contains the dual solution and the test function of the dual residual contains the primal solution. Therefore computation of residuals and weights is not well defined in the situation of coinciding discontinuities.

In this talk the problem of coinciding discontinuities in primal and dual solution is avoided by adding artificial viscosity to the dual equation, while leaving the primal problem unchanged. This procedure causes an additional residual term in the error estimator, accounting for the inconsistency between primal and dual problem.

The effectivity of the extended error estimator, assessing the global error by a suitable functional of interest, is tested numerically in 1D and 2D. Used as an indicator to control grid refinement, the proposed extended and an unmodified error estimator perform similarly. However, only the proposed modified method provides an efficient error estimator in 1D.

In 2D, with the tested parameters and approximation of weights, the effectivity index does not converge to one. However, the efficiency of the modified error estimator is better than the unmodified one.