

## An Ensemble-type Approach to Goal-oriented Error Estimation

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In this work we perform goal-oriented error estimation, error estimation with respect to a specific physical quantity of interest (goal), in an environment where the original Dual-Weighted-Residual Method (DWR) is not applicable.

In the original dual-weighted error estimation approach, the residuals of the model solution are weighted by the adjoint solution, the sensitivities of the goal towards the residuals. For environments where continuous evaluations of the residuals are impracticable, an approach has been put forward in the literature that is motivated by the Mori-Zwanzig formalism from statistical mechanics and is based on interpreting local errors as local model uncertainty. This uncertainty could then be modelled by a stochastic process that was estimated from high-resolution information.

Here, we follow along these lines of motivation and interpret the temporal fluctuations of local truncation errors (discrete residuals) stochastically by a random variable. We show how this interpretation leads to a stochastic process that parametrizes a problem-dependent temporal evolution of local truncation errors. The parameters of this stochastic process are estimated from short, near-initial, high-resolution simulations. Under the assumption that the estimated parameters can be extrapolated to the full time window of the error estimation, the resulting stochastic process is proven to be a valid surrogate for the local truncation errors.

Replacing the local truncation errors by a stochastic process puts our method within the class of ensemble methods and makes the resulting error estimator a random variable. The result of our error estimator is thus a confidence interval on the error in the respective goal. We successfully evaluate our error estimation method in a Geophysical Fluid Dynamics environment for bounded, viscous 2D Shallow-water flows.