Abstract

Nonhydrostatic correction for shallow water equations with quadratic vertical pressure distribution: A Boussinesq-type equation

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In tsunami modeling, two different systems of dispersive long wave equations are common: The nonhydrostatic pressure correction for the shallow water equations derived out of the depth-integrated 3D Reynolds-averaged Navier-Stokes equations, and the category of Boussinesq-type equations obtained by an expansion in the nondimensional parameters for nonlinearity and dispersion in the Euler equations. The first system uses as an assumption a linear vertical interpolation of the nonhydrostatic pressure, whereas the second system's derivation includes an quadratic vertical interpolation for the nonhydrostatic pressure. In this case the analytical dispersion relations do not coincide. We show that the nonhydrostatic correction with a quadratic vertical interpolation yields an equation set equivalent to the Serre equations, which are 1D Boussinesq-type equations for the case of a horizontal bottom. Now, both systems yield the same analytical dispersion relation according up to the first order with the reference dispersion relation of the linear wave theory. The adjusted model is also compared to other Boussinesq-type equations.

The numerical model with the nonhydrostatic correction for the shallow water equations uses Leapfrog timestepping stabilized with the Asselin filter and the P^1-P^1_{NC} finite element space discretization. The numerical dispersion relations are computed and compared by employing a testcase of a standing wave in a closed basin. All numerical values match their theoretical expectations.