Title

A triangular grid DG method featuring shock filters and unconditional positivity preserving time integration for shallow water flows

Abstract

Discontinuous space discretizations, especially the Discontinuous Galerkin(DG) methods, are a modern and popular class of numerical methods especially for computationally intensive fluid dynamics calculations. Their popularity is due to the fact that DG methods allow for high order approximations in combination with high flexibility – e.g. in choosing different polynomial degrees on neighbouring elements. Furthermore, due to discontinuous test and basis functions and polynomial representations within each cell, DG schemes provide rather compact stencils which facilitate an implementation in parallel.
This talk focusses on the application of the DG scheme on unstructured triangular grids to hyperbolic conservation laws such as the Euler equation of gas dynamics and the shallow water equations. In this context, due to the possible presence of discontinuities of the exact solution, such as shocks, Gibbs oscillations may affect the stability of high order approximations. For shock capturing, a special form of spectral viscosity is hence added to the semidiscrete DG scheme on triangular grids. As the viscosity term is based on the Sturm-Liouville-Operator corresponding to the DG basis, it is possible to derive so-called modal filters efficiently operating on the coefficients of the DG representation. The aim of modal filtering is to stabilize the numerical method, but by construction no effort is taken to obtain an approximation which is completely free of oscillations. Hence, in a post-processing step only carried out at output times, the remaining oscillations are removed by the image processing technique of DTV filtering.
The second part of this talk deals with the application of the DG scheme to shallow water flows with non-flat bottom topography. In this context, an extra challenge is posed by steady states that need to be preserved by well-balanced numerical methods. In addition, the DG scheme has to guarantee non-negativity of the water height. For locally refined grid obtained from a refinement at wet/dry interfaces, the stability requirements of explicit time integration lead to rather restrictive time step constraints. In this context, approaches to obtain unconditionally positivity preserving implicit time integration methods will be provided and numerically demonstrated for shallow water flow in two space dimensions.

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Dr. Sigrun Ortleb
Fachbereich Mathematik und Naturwissenschaften
AG Analysis und Angewandte Mathematik
Universitaet Kassel
Heinrich-Plett-Str. 40
D-34132 Kassel