Numerical Optimization of Geometric Energies

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Abstract:
For many applications in science and engineering, one needs trustworthy approximations of the minimizers of a given geometric optimization problem. Since the feasible set of such an optimization problem is often infinite dimensional, a common approach for the approximation of their solutions is the Ritz-Galerkin method: Computing minimizers of the energy on a finite dimensional subspace, a so-called discrete ansatz space.

Classical results, such as Cea's lemma, guarantee that the minimizers in the discrete ansatz space are arbitrarily close to the minimizers of the infinite dimensional problem if the discrete ansatz space is chosen sufficiently large---and provided that the energy is convex. Alas, many interesting geometric energies are nonconvex such that these standard results cannot be applied to prove the convergence of discrete solutions.
Moreover, it may be inefficient or even impossible to calculate the energy exactly, to use subsets of the feasible set as discrete ansatz space, or to determine discrete minimizers exactly.

In this talk, we outline a way to overcome these hardships and to reduce the convergence analysis for Ritz-Galerkin schemes to approximation theory. The developed tools will be illustrated by an application to minimal surfaces. Finally, we will realize that a priori knowledge of the regularity of minimizers is crucial for obtaining reasonable convergence rates.