Lothar Collatz Seminar
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Understanding GRP solvers

Abstract:
The generalized Riemann problem (GRP) is the Cauchy problem for a hyperbolic conservation law with piecewise smooth data that includes a jump-discontinuity. This problem naturally occurs in high-order numerical schemes: In a finite volume scheme with a reconstruction step, or in a discontinuous Galerkin finite element method, the solution of the PDE is represented at each time-step by a piecewise smooth function with possi-ble jumps at the cell interfaces. We are mainly interested in solving the GRP as a building block for high-order numerical schemes. In general, no exact solutions of the GRP for nonlinear systems are available.

Several approximate GRP solvers have been proposed, which typically employ one of the following strategies: (1) "Instantaneous interaction", which roughly speaking leadsto a Taylor approximation of the solution; or (2) "Evolution in the small", which means evolving the solution locally in each cell and computing numerical fluxes from the evolved data without resolving the full wave-pattern of the GRP. Recently, approach (2) gained in popularity, because it is a lot easier to code and can be adapted to handle stiff source terms more easily. However, a systematic comparison of the two approaches and a rigorous analysis of its theoretical properties seem to be missing in the literature. Our aim is to contribute a first step towards closing this gap.

In this talk we give a short introduction to the theory of both classical and generalized Riemann problems, see different GRP solvers in action, and take a closer look at what evolution in the small solvers are actually trying to compute from an analytical perspective. We explore the guiding principles behind the design of GRP solvers and discuss how to analyse and improve them.

For further information please contact

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www.c3s.uni-hamburg.de/news-events/seminar-c3s.html