

Errors due to discretization are inherent to the numerical solutions of Ocean General Circulation Models (OGCM). However, obtaining reliable estimates for these errors is a difficult undertaking.

Here, we show how to develop a stochastic dual-weighted error estimator for the estimation of the discretization error in physical quantities of interest (goals) that is applicable to ocean models. Towards this aim, we extend the dual-weighted error estimation technique by a stochastic process with memory. In this, we extend previous work on memory-less stochastic dual-weighted methods for two-dimensional wave-type flows.

The introduction of memory effects is the key new element of our extension and is shown to be crucial in the estimation of goal errors in an ocean model setting. The memory governs the temporal evolution of the stochastic process. We interpret the memory as a stochastic representation of physical constraints on the time-evolution of the essential building block of our stochastic dual-weighted error estimator – the local truncation error. The memory of the stochastic process is represented by temporal correlation coefficients directly or by their upper bound. The temporal correlation coefficients and other required stochastic quantities of the stochastic process are estimated from high-resolution model information at near-initial times. Our resulting stochastic dual-weighted approach is equivalent to a linearized stochastic-physics ensemble, but in contrast to the ensemble it only requires a single model integration and a single adjoint integration.

In order to study the applicability of our stochastic dual-weighted error estimator for OGCMs, we focus on important oceanic features: the presence of lateral boundaries with their associated boundary currents, and the phenomenon of baroclinic instability within a stratified ocean. Both phenomena are studied by means of idealized experiments, the Munk gyre and the flow against an island for lateral boundaries, and the spherical channel experiment for baroclinic instabilities. For flows with boundaries, we find that our stochastic error estimator provides meaningful error bounds for a range of physically relevant goals. For the eddying flow regime due to baroclinic instabilities, the stochastic process of our error estimator is modeled as a compound of a horizontal, a vertical, and a temporal structure. To be applicable in our stochastic dual-weighted error estimation framework, we propose a generalization of its temporal structure to include additional, possibly negative time-correlations.