

# Beyond numerical simulations: studying nonlinear dynamics with polynomial optimization

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Systems characterised by complex nonlinear dynamics lie at the heart of 21st century technology. Typical examples include turbulent flows, which are central to the transport and aviation industries, smart energy networks and biological systems of interest in synthetic biology. While advances in computational technology have enabled direct numerical simulations of many such systems, these have two inherent drawbacks. First, one is typically interested only in a few average quantities, such as the mean lift and drag forces on an aircraft. To compute these quantities accurately, however, one must resolve all spatio-temporal scales of the system, which often requires prohibitively large computational resources. Second, while numerical simulations offer a detailed description of the system's evolution from a given initial conditions, they cannot be used to draw conclusions that hold for all possible initial states within a given (infinite) set. Recently, an alternative approach that addresses these shortcomings has been proposed. This approach, based on a generalisation of the concept of a Lyapunov function, enables one to compute rigorous bounds on a time-averaged quantity of interest, independently of the initial condition and with no need to simulate the underlying system. For nonlinear systems with polynomial dynamics, these bounds can be optimized numerically using tools for polynomial optimization, and can be made arbitrarily sharp at the expense of increasing computational cost. In this talk I will describe this recent bounding framework in detail, and present a range of examples that demonstrate its potential. Current limitations, open problems and possible directions for future research will also be briefly discussed.