Kinetic-Induced Moment Systems for Balance Laws

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Based on the relation between kinetic theory and non-linear hyperbolic equations, we derive a kinetic-induced moment system for the spatially one-dimensional Burgers' Equation and Shallow Water Equations [1]. The derivation is based on an artificial Boltzmann-like transport equation with a BGK-relaxation [2] and its corresponding moments w_k,

$$\partial_t f(t,x,\xi) + \xi \partial_x f(t,x,\xi) = \frac{1}{\varepsilon}[f_0(Q,\xi) - f(t,x,\xi)], \qquad 0 < \varepsilon \ll 1$$

$$w_k = \int_{\mathbb{R}} \xi^k \, f(t,x,\xi) \, \mathrm{d}\xi, \qquad \text{with k} \in \mathbb{Z}_{\geq 0}$$

The resulting infinite system is a coupled system of balance laws, which depends on the relaxation parameter ε coming from the kinetic equation. Using Chapman-Enskog-like asymptotic expansion techniques [3], it will be shown that at each order of ε, a scale-induced closure is possible, which results in a finite number of moment equations. Of particular interest is the above mentioned coupled system obtained from a third order closure, which in the formal limit $\varepsilon \to 0$, yields the original system of equations with an additional variable that acts as a monitoring function of particular flow structure (like shocks and rarefaction waves) and their potential applications in the description of small-scale geophysical flows. This new unknown can be used on the one hand into the construction of adaptive numerical methods, and on the other hand as a basis to derive novel parametrization for subgrid closures.

References

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[3] H. Struchtrup, *Macroscopic Transport Equations for Rarefied Gas Flows*, Springer-Verlag, Berlin, 2005.