Finite element analysis for Free Material Optimization

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Abstract

In Free Material Optimization, for a chosen cost functional, the design variable is the full material tensor E of a body Ω governed by the laws of linear elasticity

$$\begin{aligned} -\mathrm{div}(\sigma) &= 0 && \text{in } \Omega \\ \sigma \cdot n &= f && \text{on } \Gamma \\ u &= 0 && \text{on } \Gamma_0 \\ \sigma &= Ee(u) && \text{in } \Omega \end{aligned}$$

with strain e and stress σ, and traction and displacement boundary conditions on the boundary parts Γ and Γ_0, resp. Writing the tensor in matrix notation one obtains a control-in-the-coefficients problem for the material tensor.

In this talk we give a brief introduction to Free Material Optimization and present results in the finite element analysis of the tensor identification problem, which is obtained by minimizing a Tikhonov regularized tracking-type cost functional

$$\min_E J(E, u(E, f)) = \frac{1}{2} \|u(E, f) - z\|_{L^2}^2 + \frac{\gamma}{2} \|E\|_{L^2}^2$$

for given (e.g., measured) displacement data z. We employ the variational discretization approach, where the control, i.e., the material tensor E, is only implicitly discretized. Using the concept of H-convergence we can prove convergence towards norm-minimal solutions of the continuous optimization problem.