Alexander Lohse
Universität Hamburg
Differentialgleichungen und Dynamische Systeme

Hamburg, May 2016

Heteroclinic dynamics – switching in low dimensional networks

A heteroclinic trajectory solution of a smooth dynamical system $\dot{x} = f(x)$ connects two equilibria $\xi_1, \xi_2 \in \mathbb{R}^n$ in the sense that it converges to ξ_1 for $t \to -\infty$ and to ξ_2 for $t \to +\infty$. A finite set of such equilibria and connections forming a topological circle is called a *heteroclinic cycle*, and a connected union of several cycles is a *network*. The dynamics associated with these objects have been used to model stop-and-go behaviour of applications in various fields, including neuroscience, geophysics, game theory and population dynamics.

In the context of heteroclinic networks the term *switching* refers to a particular form of complex dynamics near a network: trajectories follow any possible sequence of connections that can be prescribed given the network architecture. We consider simple heteroclinic networks in $\mathbb{R}^n$ and give sufficient conditions for the absence of a weak form of switching (i.e. along a connection that is common to two cycles), generalizing a similar result in the work of M. Aguiar (Physica D 240, 1474–1488, 2011). Moreover, we illustrate that these conditions are natural for a large class of cycles, and look at two examples of networks in $\mathbb{R}^5$ that show the complexity of the dynamics if they are broken.