

# POD-Based Economic Model Predictive Control for Heat-Convection Phenomena

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## Abstract

We consider an optimal boundary control problem subjected to linear time-dependent convection-diffusion equation together with bilateral control and pointwise state constraints. Due to the pointwise state constraints, we utilize a virtual control approach to obtain a regularized optimal control problem, which is solved by a primal-dual active set strategy (PDASS). To speed up the PDASS, a reduced-order approach based on proper orthogonal decomposition (POD) is applied and an a-posteriori error analysis ensures that the computed (suboptimal) POD solutions are sufficiently accurate. An Economic Model Predictive Control strategy is considered to treat the long-time horizon and the problem's parameters' changes.

## References

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