Identification and characterization of coherent behavior in flows

The motion of tracers in fluids flows is crucially influenced by coherent structures. Due to their strong impact on global transport and mixing processes the characterization of these Lagrangian objects is a topic of intense current research. From a probabilistic point of view, coherent sets are regular regions in the physical domain of the flow that move about with minimal dispersion.

Coherent sets can be efficiently identified via Perron-Frobenius operators (transfer operators). These linear Markov operators can be approximated within a set-oriented numerical framework based on Ulam's method. Subdominant singular values/vectors of the resulting stochastic matrices are then used to determine and characterize the structures of interest. While mathematically sound, transfer operator constructions have some computational disadvantages when studying nonautonomous systems. This has led to the recent development of data-based approaches. In this context, spatio-temporal clustering algorithms have been proven to be very effective for the extraction of coherent sets directly from given trajectories. In particular, a discrete representation of the dynamics in terms of a trajectory network forms the basis of a computationally very attractive and flexible framework, which is also applicable when the underlying data is sparse and incomplete.

In this contribution, we will review these computational approaches and apply them to a number of example systems, including turbulent Rayleigh-B\'enard convection flows.