Lower Bounds for the Advection-Hyperdiffusion equation

Fabian Bleitner, Universität Hamburg

Inspired by the work of Nobili and Pottel [1] on mixing estimates for the Advection-Diffusion equation we study the Advection-Hyperdiffusion equation on the whole space in two and three dimensions with the goal of understanding the decay of the H−1- and L2-norm in time. We view the advection as a perturbation of the hyperdiffusion equation and employ the Fourier Splitting method first introduced by Schonbek in [2] for scalar parabolic equations and later generalized to a broad class of equations including Navier Stokes equations and Magneto Hydrodynamic systems. This approach consists of decomposing the Fourier space along a sphere with radius decreasing in time. Combining the Fourier splitting method with classical PDE techniques applied to the hyperdiffusion equation we find a lower bound for the H−1-norm by interpolation.

[1] C. Nobili and S. Pottel. Lower bounds on mixing norms for the advection diffusion equation in Rd. In: Preprint at arXiv:2006.04614 (2020).

[2] Maria Elena Schonbek. Decay of solution to parabolic conservation laws. In: Communications in Partial Differential Equations 5.4 (1980), pp. 449–473. eprint: https://doi.org/10.1080/0360530800882145.