Parameter-free shape optimization: Glimpses on sensitivities, gradients and metrics for engineers

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In the last decade, parameter-free approaches to shape optimization problems have matured to a state where they provide a versatile tool for complex engineering applications. Instead of parameter-based approaches, they provide the mathematical basis for discretization-independent solutions and a rigorous definition of the space of admissible shapes. Furthermore, the analytical effort in deriving shape derivatives is frequently reduced and itself discretization independent, i.e., it can be combined with different numerical methods in a second step. To this end, the consistency of the shape derivative from parameterized approaches is lost while flexibility is gained.

However, parameter-free or continuous expressions for shape derivatives, cannot be directly used as a descent direction in gradient-based optimization strategies. Instead, a proper descent direction needs to be computed by means of an auxiliary problem, which is connected to the choice for an inner product. While several choices for these auxiliary problems were investigated in the mathematical community, the complexity of the concepts behind their derivation has prevented the engineering community from applying them in practice and taking advantage of this development. Instead, standard choices or comparatively simple smoothing approaches are applied without referencing an inner product and thus lacking a clear definition of space of admissible shapes.

This work aims to introduce several more advanced choices for inner products and give an illustrative explanation of their advantages and disadvantages. This makes them available for communities without a profound mathematical background. Further, the work provides some guidance to the state-of-the-art literature for readers interested in this background. Finally, popular choices for the auxiliary problem are compared by applying them in the context of an illustrative test case and a shape optimization problem from fluid dynamics.