Constrained shape optimization in shape spaces: Models raising from PDE to VI constraints and from deterministic to stochastic settings

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A challenge in shape optimization is in the modeling of shapes, which do not inherently have a vector space structure. If one cannot work in vector spaces, one possible approach is to cast the sets of shapes in a Riemannian viewpoint, where each shape is a point on an abstract space like a manifold—or, more generally, a diffeological space—equipped with a notion of distances between shapes.
In this talk, we first apply the differential-geometric structure of Riemannian shape spaces to the theory of deterministic shape optimization problems. Since many relevant problems in the area of shape optimization involve a constraint in the form of a partial differential equation, which contains inputs or material properties that may be unknown or subject to uncertainty, we consider also stochastic shape optimization problems. We present a novel method, which is the extension of the classical stochastic gradient method to infinite-dimensional shape manifolds. The method is demonstrated on a shape optimization problem. A motivation for our model is in electrical impedance tomography, where the material distribution of electrical properties such as electric conductivity and permittivity inside the body is to be determined. Additionally, we consider shape optimization problems constrained by variational inequalities (VI).
These problems are highly challenging in contrast to classical VIs, where no explicit dependence on the domain is given. Firstly, one needs to operate in inherently non-linear, non-convex and infinite-dimensional shape spaces. Secondly, one cannot expect the existence of the adjoint state and one cannot expect for an arbitrary shape functional depending on solutions to VIs the existence of the shape derivative or to obtain the shape derivative as a linear mapping. Thus, the problem cannot be solved directly without any regularization techniques.

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