Towards computing high-order p-harmonic descent directions and their limits in shape optimization

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Shape optimization constrained to partial differential equations is a vivid field of research with high relevance for industrial grade applications. Recent development suggests that using a p-harmonic approach to determine descent directions is superior to classical Hilbert space methods [2]. A shape is described by a part of the boundary of the computational domain which is free for deformation. For optimization algorithms the sensitivity of the cost function with respect to variations of the shape has to be represented in an appropriate space of deformations. In the p-harmonic approach, the descending deformations are obtained by the solution of a p-Laplace problem with a Neumann condition on the free boundary and homogeneous Dirichlet conditions on the remaining boundary of the computational domain.
State-of-the-art numerical algorithms for this type of problem require an iteration over the order p and break down from numerical problems when trying to archive higher orders $p > 5$. We adapt the idea of solving scalar Dirichlet problems for the p-Laplacian using interior-point methods [1] to provide an efficient algorithm for the variational problem associated to the descent directions. This enables us to perform computations without iterating the order, providing high-order solutions verified by numerical experiments for a fluid dynamic setting in 2 and 3 dimensions.
A general requirement on the resulting transformations of the computational domain is to keep it of Lipschitz type. While solutions for finite p yield approximations in $W1,p$, analytically only descent directions in $W1,\infty$ are admissible. This is challenging since the ∞-Laplace equation itself is in general not the limit of the corresponding p-Laplace equations. Further, the solutions may be non-unique due to the change of signs in the source term necessary for shape optimization. We make progress towards a novel algorithm for ∞-Laplace problems providing admissible descent directions that are the limit of the p-harmonic deformations.

References

[1] Sébastien Loisel. Efficient algorithms for solving the p-Laplacian in polynomial time. *Numerische Mathematik*, 146(2):369–400, 2020.
[2] Peter Marvin Müller, Niklas Kühl, Martin Siebenborn, Klaus Deckelnick, Michael Hinze, and Thomas Rung. A Novel p-Harmonic Descent Approach Applied to Fluid Dynamic Shape Optimization. *Structural and Mulitdisciplinary Optimization*, 2021.